MTH 201: Multivariable Calculus and Differential Equations

Homework VI

(Due 29/10)

1. Write six different iterated triple integrals for the following and evaluate one of the integrals.
   (a) The volume of the region in the first octant enclosed by the cylinder $x^2 + z^2 = 4$ and $y = 3$.
   (b) The volume of the region bounded by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.

2. Evaluate the following triple integrals.
   (a) $\displaystyle\int_0^{\pi/6}\int_0^1\int_{-2}^3 y \sin z \, dx \, dy \, dz.$
   (b) $\displaystyle\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \cos(u + v + w) \, du \, dv \, dw.$
   (c) $\displaystyle\int_0^{\pi/4}\int_0^{\ln \sec v}\int_{-\infty}^{2t} e^x \, dx \, dt \, dv.$
   (d) $\displaystyle\int_0^7\int_0^2\int_0^{\sqrt{4-q^2}} \frac{q}{r+1} \, dp \, dq \, dr.$

3. Find the following volumes using triple integrals.
   (a) The tetrahedron in the first octant bounded by the coordinate planes and the plane passing through $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$.
   (b) The region common to the interiors of the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.
   (c) The region cut from the cylinder $x^2 + y^2 = 4$ by the plane $z = 0$ and the plane $x + z = 3$.
   (d) The region between the cylinder $z = y^2$ and the xy-plane that is bounded by the planes $x = 0$, $x = 1$, $y = -1$, and $y = 1$.
   (e) The region cut in the first octant bounded by the coordinate planes, the plane $x + y = 4$, and the cylinder $y^2 + 4z^2 = 16$.

4. Evaluate the following integrals by changing the order of integration in an appropriate way.
   (a) $\displaystyle\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2} \, dy \, dx \, dz.$
   (b) $\displaystyle\int_0^2\int_0^{4-x^2}\int_0^x \frac{\sin 2z}{4 - z} \, dy \, dz \, dx.$

5. Solve for c, if:
   (a) $\displaystyle\int_0^1\int_0^{4-a-x^2}\int_a^{4-x^2-y} dz \, dy \, dx = \frac{4}{15}.$
   (b) the volume of the ellipsoid $x^2 + (y/2)^2 + (z/c)^2 = 1$ equals 8π.